Derivation of the Quadratic Equation Formula

Consider the general quadratic equation: $ax^2 + bx + c = 0$

where, a, b and c are constants

$(x+1)^2 = x^2 + 2x + 1$ Note: $(x+1)^2 = (x+1)(x+1) = x(x+1) + 1(x+1)$

$= x^2 + x + x + 1$

$= x^2 + 2x + 1$

Similarly

$(x+2)^2 = x^2 + 4x + 4$

$(x+3)^2 = x^2 + 6x + 9$

$(x+4)^2 = x^2 + 8x + 16$

$(x+5)^2 = x^2 + 10x + 25$

Summarise the above pattern in words. Do you agree with the summary: the square of the first, twice the product and the square the second.

Now, start with $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ *why?*

$x^2 + \frac{b}{a}x = -\frac{c}{a}$ *why?*

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$ *why?*

$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$ *why?*

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$ *why?*

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2-4ac}}{2a}$ *why?*

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$ *why?*

$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ *why?*