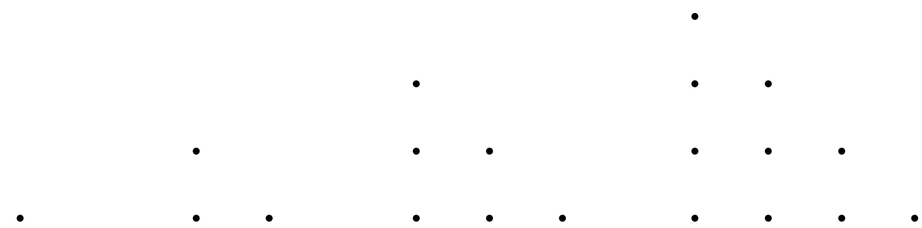
The n^{th} term Formula of the Triangular Sequence



1 3 6 10 15 etc

1^{st} difference 2 3 4 5 etc

2^{nd} difference 1 1 1 etc

Since the 2^{nd} difference is constant the expression of the n^{th} term must be quadratic.

So, expect $an^2 + bn + c$ generally, (1)

Now, 1^{st} term: $a + b + c = 1$ (2)

2^{nd} term $4a + 2b + c = 3$ (3)

Subtract equation (2) from equation (3) to get:

$$3a + b = 2 \qquad (4)$$

3^{rd} term: $9a + 3b + c = 6$ (5)

Subtract equation (2) from equation (5) to get:

$$8a + 2b = 5 \qquad (6)$$

Now, solve equations (4) and (6) simultaneously:

$$\begin{array}{r|} 3a + \ b \ = 2 \\ 8a + 2b = 5 \end{array}$$

Or $\begin{array}{r|} 6a + 2b = 4 \\ 8a + 2b = 5 \end{array}$ Why?

So, $2a = 1$ Why?

and $a = \frac{1}{2}$ Why?

Now, substitute this value of *a* in equation (4)

$$3a + b = 2$$
$$3 \times \frac{1}{2} + b = 2$$
$$b = \frac{1}{2} \qquad \text{why?}$$

and since $a + b + c = 1$ (from 2)

$$c = 0$$

Substituting these values in (1):

$$\frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}n(n + 1)$$

Now, let us check if this formula works for, say, the 4^{th} term: $\frac{1}{2} \times 4(4 + 1) = 10$. Yes it does!

Check if the formula works for other terms.