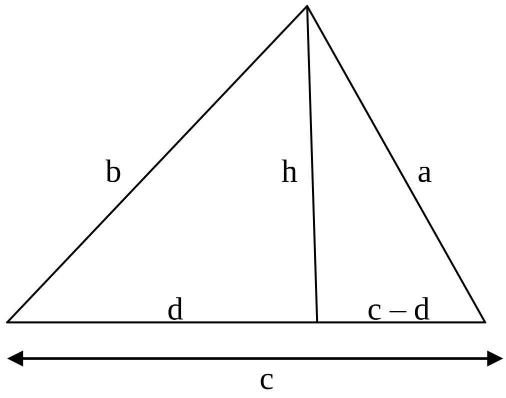
Proof of Heron's Formula

Using Pythagoras Theorem



$$b^2 = d^2 + h^2 \qquad (1)$$

$$a^2 = h^2 + (c-d)^2 \qquad (2)$$

$$a^2 - b^2 = (c-d)^2 - d^2 \qquad why?$$

$$a^2 - b^2 = c^2 - 2cd + d^2 - d^2 \qquad why?$$

$$a^2 - b^2 = c^2 - 2cd$$

$$-a^2 + b^2 = -c^2 + 2cd \qquad why?$$

$$-a^2 + b^2 + c^2 = 2cd \qquad why?$$

$$d = \frac{-a^2 + b^2 + c^2}{2c} \qquad why?$$

From (1):

$$h^2 = b^2 - d^2 \qquad why?$$

$$= \left(\frac{2bc}{2c}\right)^2 - \left(\frac{-a^2 + b^2 + c^2}{2c}\right)^2 \qquad why?$$

$$= \frac{(2bc - a^2 + b^2 + c^2)(2bc + a^2 - b^2 - c^2)}{4c^2} \qquad why?$$

Hint: $(a \pm b)^2 = a^2 \pm 2ab + b^2$

$$= \frac{\left[(a+c)^2 - a^2)\right]\left[(a^2 - (b-c)^2\right]}{4c^2} \qquad why?$$

$$= \frac{(b+c-a)(b+c+a)(a+b-c)(a-b+c)}{4c^2} \qquad why?$$

$$= \frac{2\left[\frac{b+c+a-2a}{2}\right] \times 2\left[\frac{a+b+c}{2}\right] \times 2\left[\frac{a+b+c-2c}{2}\right] \times 2\left[\frac{a+b+c-2b}{2}\right]}{4c^2}$$

$$why?$$

$$= \frac{2\left[\frac{b+c+a}{2} - a\right] \times 2\left[\frac{a+b+c}{2}\right] \times 2\left[\frac{a+b+c}{2} - c\right] \times 2\left[\frac{a+b+c}{2} - b\right]}{4c^2}$$

$$= \frac{2(s-a) \times 2s \times 2(s-c) \times 2(s-b)}{4c^2} \qquad why?$$

$$= \frac{4s(s-a)(s-b)(s-c)}{c^2} \qquad why?$$

$$\therefore \ h = \frac{2}{c} \times \sqrt{s(s-a)(s-b)(s-c)} \qquad why?$$

But the area of a triangle is:

$$A = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} ch$$

$$= \frac{1}{2} c \times \frac{2}{c} \times \sqrt{s(s-a)(s-b)(s-c)}$$

$$\therefore A = \sqrt{s(s-a)(s-b)(s-c)} \qquad why?$$

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as required