# Proof of the Mortgage Formula

In the mortgage calculations a series sum formula similar to the one below was used without proving it.

$$\sum_{k=1}^{n}\frac{1}{(1+x)^k}=\frac{1}{(1+x)}+\frac{1}{(1+x)^2}+\frac{1}{(1+x)^3}+\ldots+\frac{1}{(1+x)^n}=\frac{1}{x}\left[1-\frac{1}{(1+x)^n}\right] \qquad (1)$$

To prove the above relationship the geometric series sum formula needs to be proved first. The geometric series sum formula will then be utilised.

Let $S_n$ be the sum of *n* terms of the geometric series below:

$$S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1} \qquad (2)$$

$$rS_n = \quad ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \qquad (3)$$

Equation 3 is *r* times equation 2.

Now, subtract equation 3 from equation 2 to get

$$S_n(1-r) = a(1-r^n)$$

Or, $$S_n = \frac{a(1-r^n)}{1-r} \qquad (4)$$

Note that *a* is the first term of the series and *r* is the common ratio (the result of division of two consecutive numbers of the geometric series in equation 2 and 3 above).

Now, the first term of the geometric series in equation 1 is $\frac{1}{1+x}$ and the common ratio is $\frac{1}{1+x}$

So, substituting for the first term and the common ratio in equation 4 will give:

$$S_n = \frac{\frac{1}{1+x}\left[1-\frac{1}{(1+x)^n}\right]}{1-\frac{1}{1+x}} = \frac{1}{x}\left[1-\frac{1}{(1+x)^n}\right] \qquad why? \qquad (5)$$

Finally, since *x* is a working variable one can replace *r* for *x*. That completes the proof.