

Compound Interest Formula

Show that the formula for calculating compound interest is give by:

$$A_n = P(1 + r)^n$$

Where A_n is the amount at the end of n^{th} year

P is the principal (the initial investment)

r is the interest rate expressed as decimal (i.e., $r = \frac{R}{100}$)

R is the nominal interest rate

n is the number of investment years

Let A_1 be the amount at the end of 1st year.

The interest at the end of the 1st year is Pr

So the amount at the end of the 1st year is:

$$A_1 = P + Pr \quad (\text{initial investment and interest earned})$$

Or
$$= P(1 + r)$$

At the beginning of the 2nd year A_1 will be invested to earn interest at the same rate r .

So at the end of the 2nd year the interest earned is A_1r and the total amount is:

$$A_2 = A_1 + A_1r \quad (\text{capital at the end of the first year and interest earned at the end the 2nd year})$$

Or
$$= A_1(1 + r) \quad \text{why?}$$

Or
$$= P(1 + r)(1 + r) \quad \text{why?}$$

Or
$$A_2 = P(1 + r)^2 \quad \text{why?}$$

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So,
$$A_n = P(1 + r)^n \quad \text{as require, why?}$$